Heavy quark mass dependence of semileptonic form factors for B decays*

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We present our study of the dependence of the heavy-to-light semileptonic B decay form factors on the heavy-light meson mass M_{PS}. Simulations are made over a range of the heavy quark mass covering both the charm and bottom quarks using the $O(a)$-improved clover action at $\beta = 5.9$ on a $16^3 \times 40$ and $24^3 \times 64$ lattice. We find that a weak dependence of form factors on M_{PS} observed in previous studies in the region of charm quark persists up to the region of b quark. The soft pion relation $f^0(q^2_{max}) = f_B/f_\pi$ is examined and found to be largely violated.

1. Introduction

In spite of their small branching fraction, the exclusive semileptonic decays $B \to \pi l\nu$ and $B \to \rho l\nu$ are expected to become important processes to determine V_{ub}. A model independent calculation of the form factors relevant for these decay processes is possible using lattice QCD, and several attempts have already been made[1–4]. To avoid problems associated with large heavy quark mass m_Q, however, these studies made simulations in the charm quark mass region and extrapolated results to the b quark.

In this report we describe our examination of the heavy quark mass dependence of the form factors. For this purpose we employ the $O(a)$-improved clover action in the formalism of the FNAL group[5], and carry out simulations over a wide range of heavy quark mass including the bottom as well as charm quark masses[6]. Our results for f_B with this formalism show that systematic errors due heavy quark are small enough for the b quark[7], and we expect the systematic error to be also well under control for the form factors.

*presented by S. Tominaga

2. Simulation

We use the plaquette action and the clover action with the clover coefficient determined at the one-loop level[7]. Measurements are made at $\beta = 5.9$ with 100 configurations on a $16^3 \times 40$ lattice and 110 configurations on a $24^3 \times 64$ lattice. The lattice scale is set by the ρ meson mass which gives $a^{-1} = 1.64(2)$ GeV. Six values of the heavy quark hopping parameter in the range $\kappa_h = 0.0718 - 0.1245$ are employed. The chiral limit is taken for the light quark with results obtained for the light quark hopping parameter in the range $\kappa_l = 0.13630 - 0.13816$ to the critical value $\kappa_c = 0.13901(1)$.

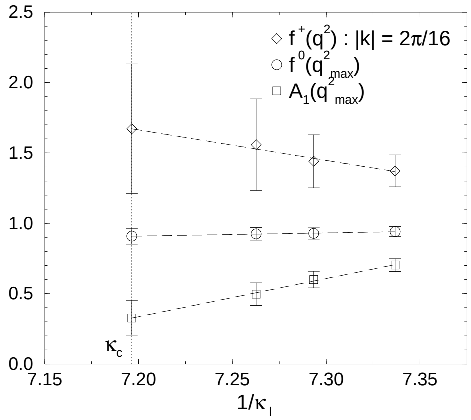

Figure 1. Chiral extrapolation of $f^+(q^2)$ for minimum pion momentum, $f^0(q^2_{max})$ and $A_1(q^2_{max})$ obtained near B mass region ($\kappa_h = 0.0973$) for $16^3 \times 40$ lattice.

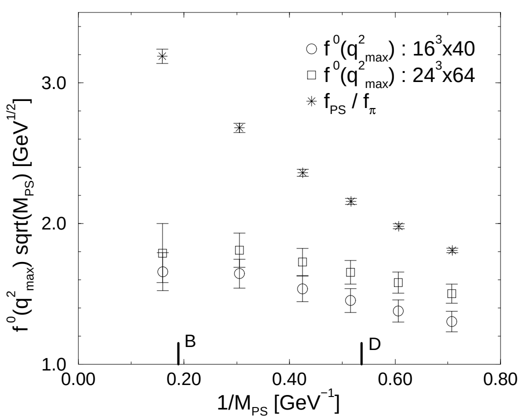

Figure 2. $f^0(q^2_{max})\sqrt{M_{PS}}$ as a function of $1/M_{PS}$. Circles correspond to data on a $16^3 \times 40$ lattice and squares on a $24^3 \times 64$ lattice. Crosses represent $f_{PS}\sqrt{M_{PS}}/f_\pi$.

The form factors for the B to π decay are obtained from the heavy-to-light three-point function $\langle P_{HL}(T/2)V_\mu(t)P^\dagger_{LL}(0)\rangle$ for the vector current $V_\mu(t)$ by dividing out the appropriate two-point functions. The pseudo scalar fields $P_{HL}(T/2)$ and $P_{LL}(0)$ are smeared with the measured wave function obtained in Ref. [7]. The B meson is taken at rest ($|\vec{p}|$=0) and the pion is given up to a unit of momentum ($|\vec{k}|$=0, $2\pi/L$), above which the signal becomes unacceptably noisy. The form factors for the B to ρ decay is obtained in a similar manner, for which we consider only the zero-recoil decay $|\vec{p}| = |\vec{k}| = 0$.

We do not include the perturbative Z factor for the currents in this study, since results for finite heavy quark masses are not fully available.

3. Results

In Fig. 1 we plot $f^+(q^2)$, $f^0(q^2_{max})$ and $A_1(q^2_{max})$ as a function of $1/\kappa_l$ for $\kappa_h = 0.0973$ which is near the b quark mass. Results for f^+ are for the minimal non-zero pion momentum. We observe that the dependence in $1/\kappa_l$ is quite small for $f^0(q^2_{max})$. In contrast there is a clear slope for $A_1(q^2_{max})$, which differs from previous results[2,3]. For the chiral extrapolation we adopt a linear form in $1/\kappa_\ell$ as shown in Fig. 1, with which our results are consistent.

Heavy quark symmetry predicts that $f^0\sqrt{M_{PS}}$, $f^+/\sqrt{M_{PS}}$ and $A_1\sqrt{M_{PS}}$ scales toward the heavy quark mass limit. In Fig. 2 we plot $f^0(q^2_{max})\sqrt{M_{PS}}$ in the chiral limit as a function of $1/M_{PS}$. We observe that a small $1/M_{PS}$ correction, as suggested by results of previous studies performed around the charm quark mass[1,3,4], also holds in our data. Furthermore the weak dependence persists to the region of b quark mass. We also note a volume effect of about 10% between $16^3\times40$ and $24^3\times64$ lattices, which is statistically significant toward light meson masses.

In the chiral limit, an application of the soft pion technique predicts the relation $f^0(q^2_{max}) = f_{PS}/f_\pi$[8]. In Fig. 2 we also plot $f_P\sqrt{M_{PS}}/f_\pi$ (crosses) obtained in Ref. [7]. There is a significant discrepancy with $f^0(q^2_{max})\sqrt{M_{PS}}$, particularly toward heavy quark masses. Possible origins of the discrepancy are (i) subtleties in the chiral extrapolation of $f^0(q^2_{max})$ since q^2_{max} changes with $1/\kappa_\ell$, (ii) systematic errors due to heavy quark including corrections from the Z factor, (iii) scaling violation and breaking of chiral symmetry for light quark. It is not clear at present if these could account for the large difference seen in Fig. 2.

The $1/M_{PS}$ dependence of $f^+(q^2)/\sqrt{M_{PS}}$ is shown in Fig. 3. It is negligible for this quantity.

For f^0 and f^+ the slope in the chiral extrapolation is almost independent of the heavy quark

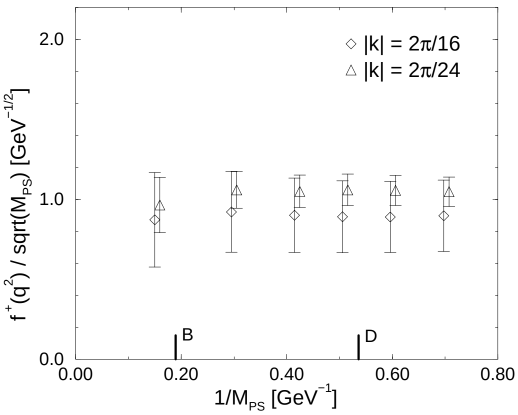

Figure 3. $f^+(q^2)/\sqrt{M_{PS}}$ in the chiral limit as a function of $1/M_{PS}$. Triangles correspond to the results on a $16^3 \times 40$ lattice and diamonds on a $24^3 \times 64$ lattice. B meson is at rest and pion carries one unit of momentum.

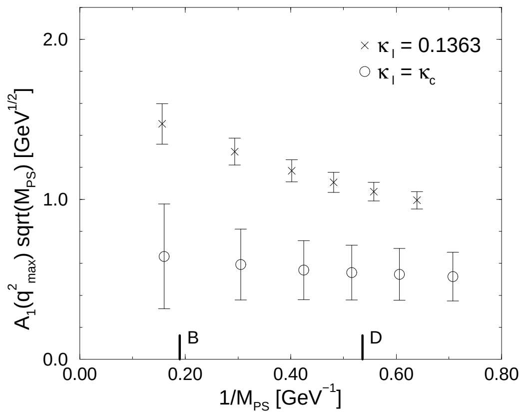

Figure 4. $A_1(q^2_{max})\sqrt{M_{PS}}$ on a $16^3 \times 40$ lattice at $\kappa_l = 0.1363$ and κ_c as a function of $1/M_{PS}$.

mass. Therefore the $1/M_{PS}$ dependence of these form factors is almost unchanged by the chiral extrapolation, as was seen in Ref. [3]. On the other hand, the large slope for A_1 shown in Fig. 1 becomes smaller as the heavy quark becomes lighter. As a result the $1/M_{PS}$ behavior of A_1 depends significantly on the light quark mass as shown in Fig. 4, where we plot results at a finite light quark mass (crosses) together with that in the chiral limit(circles).

4. Conclusions

Previous studies[1,3,4] performed on the charm quark mass region suggested that the $1/M_{PS}$ dependence of the form factors is small contrary to the case of the heavy-light decay constant which varies significantly between the static limit and the region of charm quark. Our calculation confirms this trend and extends it to the region of b quark. This is particularly puzzling for f^0 for which we find a large violation of the soft pion relation $f^0(q^2_{max}) = f_B/f_\pi$. A weak $1/M_{PS}$ dependence and a discrepancy from the soft pion relation are also observed if non-relativistic QCD is employed for heavy quark[9]. We feel that further understanding of the heavy quark mass dependence is required for a reliable calculation of heavy-to-light form factors from lattice QCD.

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